

Math 55 Quiz 8 DIS 106

Name: _____

18 Apr 2022

1. Find the solution to the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$ with initial conditions $a_0 = 2, a_1 = 1$. [5 points]

2. The characteristic polynomial to the recurrence relation is $x^2 = 7x - 10$, which has solutions $x = 2, 5$. Hence the general solution is $a_n = C_1 \cdot 2^n + C_2 \cdot 5^n$.

From the initial conditions, we know that

$$\begin{aligned}C_1 + C_2 &= 2 \\2C_1 + 5C_2 &= 1\end{aligned}$$

Hence $C_1 = 3, C_2 = -1$, and the solution is $a_n = 3 \cdot 2^n - 5^n$.

3. Decrypt the message 11 using the RSA cryptosystem with key $(5 \cdot 7, 5)$. (Your answer should be a number between 0 and 34.) [5 points]

Here $p = 5, q = 7$, so $(p - 1)(q - 1) = 24$. $5 \cdot 5 = 25 \equiv 1 \pmod{24}$, so we can take $d = 5$.

To decrypt 11, we have to compute 11^5 . This can be done using fast modular exponentiation:

$$11^2 = 121 \equiv 16 \pmod{35}$$

$$11^4 \equiv 16^2 = 256 \equiv 11 \pmod{35}$$

$$\text{So } 11^5 = 11 \cdot 11^4 \equiv 11 \cdot 11 = 121 \equiv 16 \pmod{35}.$$